Single-Parameter Adaptation for Acquisition and Tracking

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Abstract. An adaptive regulator with a smoothly variable high-order linear response controlled by a single real parameter, or a regulator including a single nonlinear element, is employed in an acquisition-tracking system. The requirement for good tracking is not compromised by the requirement for fast acquisition, and vice versa. The minimum phase character of the return ratio is preserved in a robust manner. Using intermediate design requirement simplifies the design procedure. A nonlinear static function is used as the exponent of a link transfer function. The transition between the modes is smooth and rapid. The transient responses are fast and without substantial overshoots.

1. Acquisition and tracking

Acquisition and tracking systems, like those used in homing missiles, are designed to operate in two modes: acquisition mode when the error is initially large and gradually diminish, and tracking mode when the error is maintained small. An example of the acquisition/tracking type is a homing control system for pointing a spacecraft-mounted camera, in which a rapid retargeting maneuver is followed by a slow precise scanning pattern to form a mosaic image. Another example is clock acquisition in the phase-locked loops of telecommunication systems and frequency synthesizers.

Since optimizing of an LTI controller for the purpose of acquisition and for the purpose of tracking results in quite different systems, performance of LTI controllers cannot be best both for acquisition and tracking, and the controller design involves some compromises. The trade-offs can be better resolved in nonlinear and LTV controllers.

Some of industrial acquisition/tracking controllers are very sophisticated (like those for some hard disk drives) and are designed by teams of advanced level professionals. However, large investments in the control design not always can be justified, and many practical acquisition/tracking systems use rather primitive single-parameter adaptive gain-scheduling and nonlinear regulators.

For example, a single switch in a National Semiconductor PLL IC shifts the loop response between acquisition and tracking modes by one octave thus reducing the acquisition time almost twice. The shape of the loop response is kept constant. With the chosen response shape and shifting the response not more than by one octave the switching does not cause violent transient. The chosen shape of the response is close to that optimal for tracking but not for acquisition. The acquisition, therefore, could be further improved with a better controller. (An attempt to switch directly from the response well suitable for the initial stage of the acquisition to that for the tracking causes, however, violent transients which may de-acquisit the target.)

The present paper describes certain means for making an acquisition/tracking controller with single-parameter adaptation both efficient and simple. The major performance limitations are expressed in frequency-domain: the feedback bandwidth and the robust performance are limited by the Bode integrals in conjunction with, in the acquisition mode, the plant structural modes and the sampling frequency, and in the tracking mode, with the sensor noise and the jitter sources. The tracking response need not be compromised in order to meet the requirements for good acquisition, and vice versa; these responses are also not compromised by the requirement to the adaptation transients smoothness. The last requirement together with the requirements to the rate of adaptation to be fast is addressed by designing appropriate intermediate responses and specifying the rate for changing the responses.

Regulation of the frequency responses can be performed with FIR filters. However, using the filter for gradually changing the responses requires generation of multiple commanding profiles for the filter coefficients. A simpler alternative to this option is single-parameter adaptation.

2. Loop responses

In acquisition regime the error signal is large as illustrated in Fig 1(a). In this case the controller

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should respond rapidly, i.e. the feedback bandwidth should be wide and the closed-loop response is close to that of a Bessel filter. In the acquisition mode it is not necessary however that the feedback be very large, since the error is big anyway. In contrast, in the tracking regime, the feedback bandwidth needs to be reduced to reduce the output effects of the sensor noise, but the value of the feedback at lower frequencies should be made large to reject the jitter and to make the tracking precise. The loop frequency responses for the two modes of operation are depicted in Fig. 1(b). We assume here that the feedback bandwidth in the system is limited by a structural mode with large phase uncertainty, and therefore the mode needs to be gain-stabilized. The shapes of the loop responses are optimized under the Bode integral limitations (causality limitations).

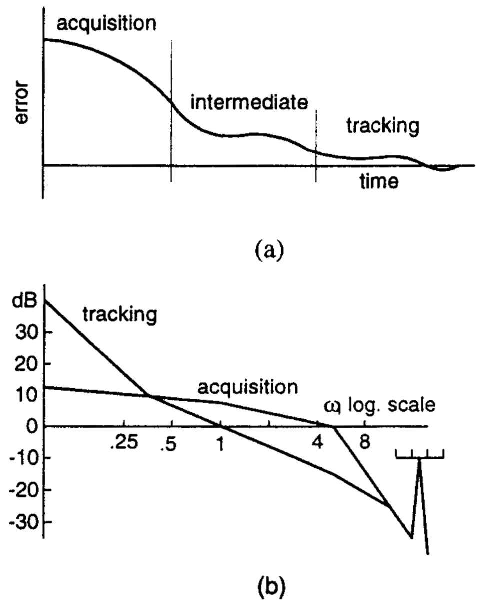


Fig. 1 (a) Error time-history and (b) acquisition and tracking loop frequency responses

The transition between the responses can be done by switching or regulating w or by using nonlinear windows: the small errors are directed to the tracking compensator, and the large errors directed into the acquisition compensator.

The intermediate combined frequency responses of the parallel channels might result in an unstable system, or in a system with small stability margins and, therefore, producing large amplitude transient responses. Special care must be taken to ensure that the intermediate response are acceptable.

This can be done with a controller using also a specified intermediate frequency response during the transition, and/or by a controller that changes the response continuously in a specified manner. In the latter case it is desired to make the response continuously optimal, i.e. providing signal to the actuator just enough for the actuator saturation, while resolving at each time the Bode trade-off between the value of feedback and the available feedback bandwidth. The continuously changing Nyquist diagram should all the time not to include the critical point and provide stability margins for the system robustness and good transient responses.

3. Linear combination of responses

The determination of the optimal frequency responses for the acquisition mode and for the tracking mode is straightforward. However, guaranteeing a smooth and fast transition from acquisition to tracking is not trivial. In improperly designed systems, the transition can generate large transients in the output and error signals, and the target can be de-acquired.

The adaptation via adjustments of a single real parameter w can use linear combination of two responses, so that the total loop response be the weighted sum of the acquisition and tracking responses:

$$W(w) = (1-w)W_{acq} + wW_{tr}, \qquad (1)$$

so that $W(0) = W_{acq}$ and $W(1) = W_{tr}$, and w smoothly varies from 0 to 1 as the transition from acquisition mode to tracking mode occurs.

Due to different slope of the Bode diagrams, in accordance with Bode phase-gain relations the difference in phase shift between the two responses is significant. At low frequencies, over some frequency range $[f_1,f_2]$, this difference exceeds π.

During the transition from acquisition to tracking, the acquisition gain response sinks and the tracking gain response rises in Fig. 1(b). For certain values of w, the gains in the two paths are equal at a frequencies within the interval $[f_1,f_2]$, and the result is that a zero of the total transfer function W transgresses into the right half-plane of s [4,5] and the system becomes unstable. The transient generated while the system remains in these states can be violent and disruptive, even causing the target to be lost.

The general conditions for the composite parallel path transfer function to become nonminimum phase when each of the channels is minimum phase, is given in [4,5]. According to this condition, when blending two linear controllers, their responses should not differ as much as those shown in Fig. 1. Hence, the controller using linear combination of two responses, (although substantially better than an LTI controller, like the beforementioned PLL controller), provides only a small range of smooth regulation and therefore precludes implementing the best possible responses for acquisition and for tracking, where the difference between these two responses is large.

4. Bilinear single-parameter regulation

In a linear system, regulation of a transfer function W using a single variable element with transfer function w is generally expressed as a bilinear function

$$W(w) = \frac{w_1 W(0) + wW(\infty)}{w_1 + w} \qquad (2)$$

where w_1 is a system function. For example, if w represents the variable impedance of a two-pole, w_1 is the driving point impedance between the terminals to which the two-pole w is connected. If w designates the transfer coefficient of an amplifier, then $-1/w_1$ is the feedback path transmission coefficient for this amplifier. Using (2) for gradual transition from acquisition to tracking instead of (1) allows to accommodate the best possible acquisition and tracking responses while keeping the intermediate responses acceptable and, in fact, being rather close to the optimal.

With an appropriate parameter (or transfer function) w_1, (2) allows to independently specify three responses: the initial, the final, and some intermediate, as compared to two responses (the initial and the final) when using (1). To achieve the widest range of smooth regulation between the acquisition and the tracking responses while keeping the transfer function minimum phase with sufficient margins, it is appropriate to use a symmetrical regulator.

5. Symmetrical regulator

The regulation is symmetrical with respect to the nominal value w_o of the variable parameter w when the maximum relative deflections of w from w_o, up and down, cause symmetrical in gain and phase variations in W as shown for gain in Fig. 2, i.e. when the regulation has the following property [3,5]:

$$Q = \frac{W(\infty)}{W(w_o)} = \frac{W(w_o)}{W(0)}. \qquad (3)$$

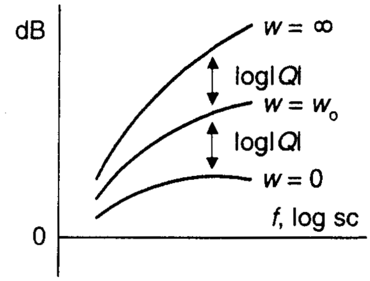


Fig. 2 Regulation frequency responses of a symmetrical regulator

By substituting (3) into (2) we have

$$w_o = w_1/Q, \qquad (4)$$

and

$$W = W(w_o)\frac{1+(w/w_o)Q}{(w/w_o)+Q}. \qquad (5)$$

The gain of the regulator expresses as

$$20\log|W| = 20\log|W(w_o)| + 20\log\left|\frac{1+(w/w_o)Q}{(w/w_o)+Q}\right| \qquad (6)$$

When $w = w_o$, the second component of (6) is 0. The second component retains the value but changes the sign when w_o is switched from 0 to ∞ as illustrated in Fig. 2. These two responses signify the range of the regulation. In the Taylor expansion of (6) the linear term dominate and the regulator gain depends on w/w_o monotonously and nearly linearly over the regulation range wider than 20 dB.

The regulator can be used for compensating the effects of the plant parameter variations in adaptive systems, and for resolving the trade-offs between the available disturbance rejection and the output noise in different regimes of operation.

The regulator responses with $W(w_o) = Q = 1/s$ are exemplified in Fig. 3. With the increase of w the plot gradually changes from that of a double integrator to a constant gain response. Correspondingly, the phase lag decreases from π to 0. It is seen that using such regulator, smooth gradual single-parameter regulation can be performed over large ranges of gain and phase shift, more than sufficient for the purpose of gradual transition from acquisition to tracking responses.

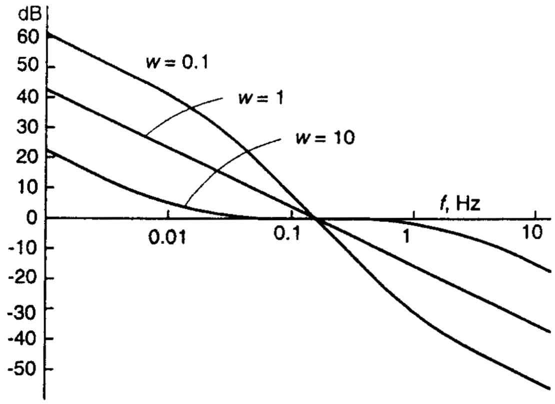


Fig. 3 Regulator responses $W(w)$ for $w_0 = 1$, $W(w_0) = 1/s$, $Q = 1/s$

Four simple block diagrams implementing the bilinear relation (4) are shown in Fig. 4. Each diagram incorporates a feedback loop and a feedforward path.

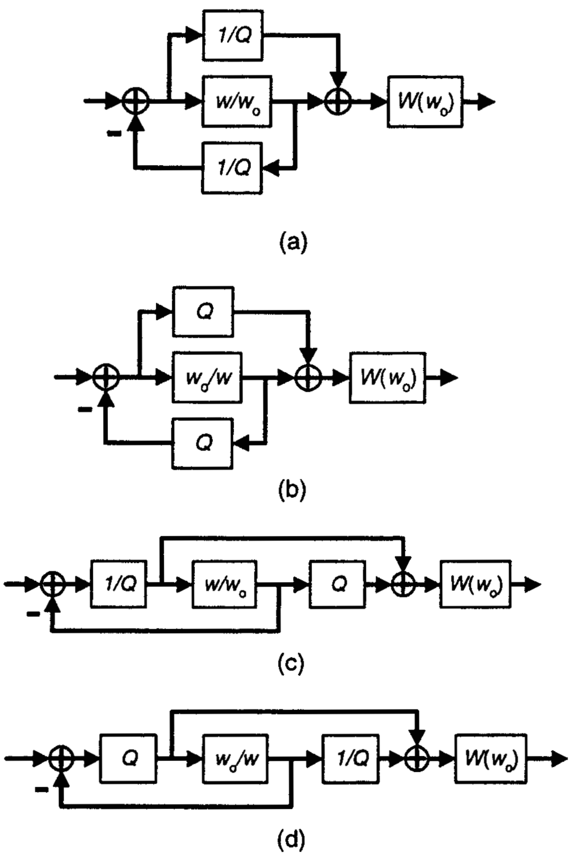


Fig. 4 Flowcharts for a symmetrical regulator

When used in a feedback system controller, the regulator can be employed with a linear gain coefficient w/w_0 variable by some adaptation or other algorithm, thus making the controller a LTV adaptive system. This is a gain-scheduling system with only a single real gain coefficient to be scheduled.

In fact, using the symmetrical regulator separates the design of a controller with gradually changing responses into three steps: (a) defining the acquisition and the tracking responses, (b) shaping the loop response for all intermediate responses, and (c) defining the rate of changing the response. These subproblems are solved sequentially thus making the design structural, and greatly simplifying the approach.

6. Nonlinear dynamic regulator with a single nonlinear nondynamic link

Instead of using an adaptive LTV system with a specific algorithm for changing a linear coefficient w, the system can be modified by replacing the variable element w by a nonlinear non-dynamic link. The resulting regulator makes a nonlinear dynamic link. Such nonlinear dynamic compensators (NDC) are discussed in [4,5]. The compensators can be analyzed and designed with describing function (DF) approach. The describing function of the entire regulator is calculated by substituted into (5) the describing function of the nonlinear element. The error resulting from DF approximation is relatively small and will be further reduced at the design stage of the system simulation and fine-tuning.

The transfer function G from the regulator input to the input of the nonlinear element is:

$$\frac{1}{1+(w/w_0)/Q} \quad \text{for the diagram in Fig. 4(a),}$$

$$\frac{1}{1+(w_0/w)Q} \quad \text{for the diagram in Fig. 4(b),}$$

$$\frac{1/Q}{1+(w/w_0)/Q} \quad \text{for the diagram in Fig. 4(c),}$$

$$\frac{Q}{1+(w_0/w)Q} \quad \text{for the diagram in Fig. 4(d)}$$

so that the signal at the nonlinear element input is $G(E)$. The describing function for the nonlinear element is therefore a function of $|G(E)|$, and the describing function of the regulator can be found by substituting this describing function into (5).

Two requirements should be considered while choosing the configuration of the NDC (from the four in Fig. 4) and the type of the nonlinear element: the system stability and good transient responses to commands and disturbances.

The system stability can be verified with iso-w describing functions which are the describing function responses with constant value of the signal amplitude at the input to the nonlinear element. Using iso-w Bode diagrams is a convenient way to assure the system stability since the iso-w transfer functions cover the same set of transfer functions as the set of the describing functions. If no such response presents instability conditions for all possible values of w, the system will have no limit cycle.

We make a conjecture now that the convergence from acquisition to tracking should be close to the fastest possible if the describing function of the nonlinear element has at all instants the value that produces the loop response which is optimal for the current value of the feedback error. In other words, these must be the conditions for the settling time be the smallest. Each of these responses are considered limited by the high-frequency properties of the plant and has the same high-frequency asymptote.

To achieve this goal, one has to try the available options in the regulator configurations and in G. If none of these function suits a specific application, the link w can be replaced by the three cascaded links L_1, w, $1/L_1$, with an appropriate linear link L_1, to change the level of the signal at the input to the nonlinear link without changing the iso-w responses.

7. The acquisition/tracking regulation response

The response for the regulation function 20 log|Q| for the acquisition/tracking problem can be find from Fig. 1, as a half of the difference in dB between the acquisition and the tracking responses. The phase stability margins for acquisition and for tracking should be at least 45o in order for the overshoot to be less than 40%, since this is a homing system and such methods of overshoot reduction as using a prefilter or command feedforward are not applicable here.

The picture is redrawn in Fig. 5 for a particular example, where the crossover frequency for tracking $f_{b\,t} = 1$ and the crossover frequency for acquisition $f_{b\,ac} = 5$. The nominal loop gain response which is the average between these two responses is shown by the dashed line.

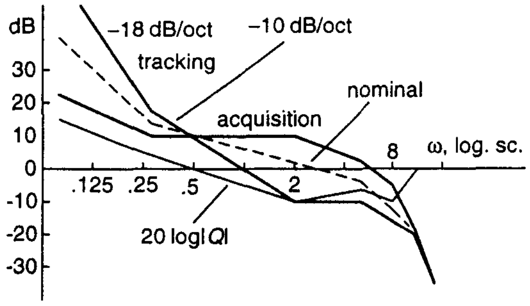


Fig. 5 Asymptotic gain responses for acquisition T_o/Q, tracking T_oQ, and regulation Q^2

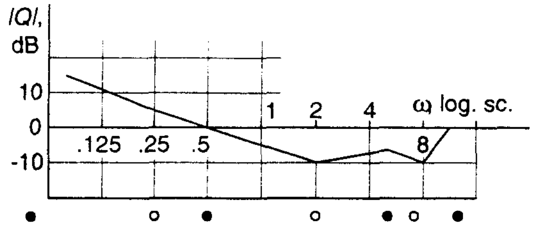


Fig. 6 Regulation asymptotic Bode diagram

The asymptotic regulation gain response is shown by a thin line. The response has a fixed point at $f \approx 0.5$, and it "rotates" about this point.

The asymptotic response of Q is redrawn in Fig. 6. It can be approximated by a response with poles and zeros placed as indicated.

The transfer function $Q(s)$ fitting this asymptotic gain response having the poles and zeros as indicated in Fig. 6, is

$$Q(s) = \frac{1.1(s+0.25)(s+2)(s+3)(s^2+5s+64)}{s(s+0.5)(s+7)(s^2+10s+100)}$$

The regulation function Q is plotted in Fig. 7.

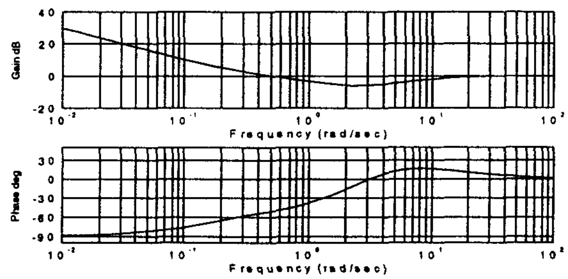


Fig. 7 Frequency response of regulation function Q

The nominal loop response is well approximated by

$$T_o(s) = \frac{250(s+0.3)}{s(s+0.005)(s^2+10s+100)}$$

as shown in Fig. 8.

The iso-w Bode diagrams for the loop transfer function

$$T = T(w_o)\frac{1+(w/w_o)Q}{(w/w_o)+Q}$$

are shown in Fig. 9. The response with maximum gain at lower frequencies (with $w = \infty$) is $T(w_o)Q$, the minimum low-frequency gain response (with $w = 0$) is $T(w_o)/Q$. The plotted responses relate to w equal to 0.01, 0.1, 1, 10, 100.

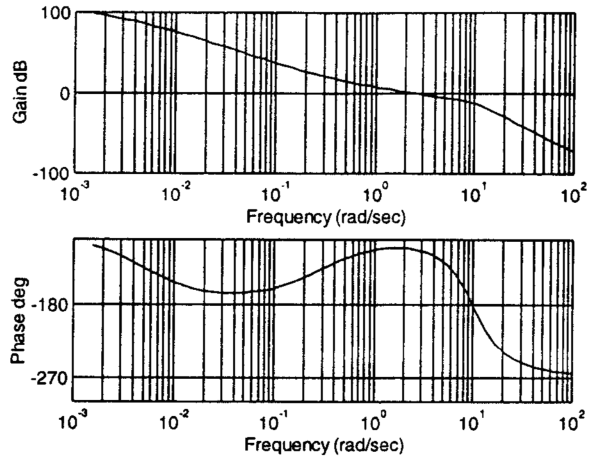


Fig. 8 Bode diagrams for return ratio T_o.

It is seen that, as desired, the set of the responses has a node at approximately 0.5 rad/sec and the gain gradually changes from the response suitable for acquisition (the one with low gain, wide bandwidth) to the response suitable for low-noise, high-jitter-rejection tracking (the one with high gain, low bandwidth). It is seen that the system is stable with any of the set of responses plotted in Fig. 9.

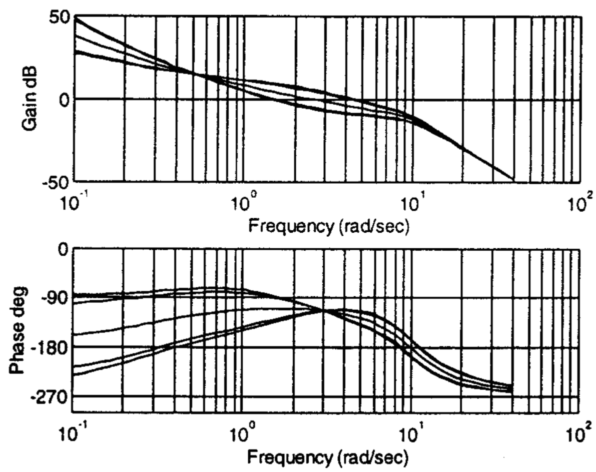


Fig. 9 Smooth loop regulation; return ratio with $w = 0.01; 0.1; 1; 10; 100$; larger w corresponds to larger crossover frequency; larger w is suitable for the acquisition, smaller w for tracking

The frequency responses of the closed-loop homing system transfer function $T/F = M$ are plotted in Fig. 10. It is seen that the tracking (closed-loop) bandwidth decreases approximately 3 times from acquisition to tracking, while preserving the same high-frequency asymptote and having a fixed point at approximately 15 rad/sec.

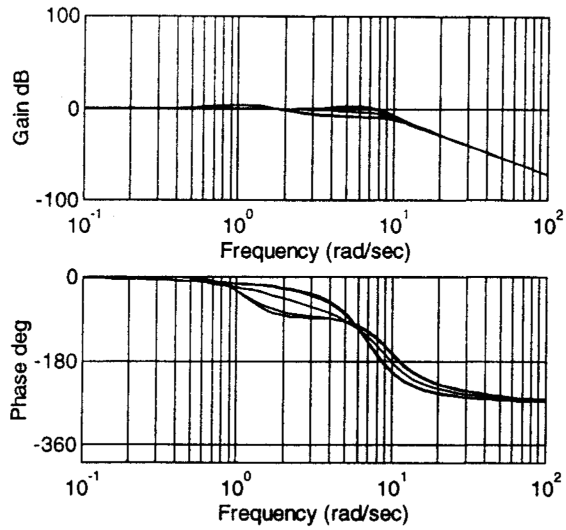
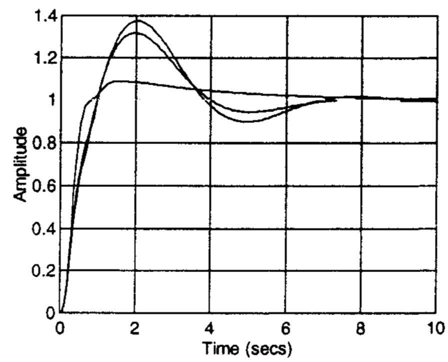
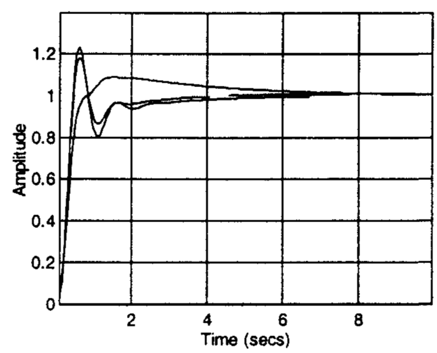


Fig. 10 Closed-loop response with $w = 0.01; 0.1; 1; 10; 100$; the bandwidth increases with w.

The closed-loop transient responses are plotted in Fig. 11. The rise-time decreases with w.



(a)



(b)

Fig. 11. Transient response of the linear homing system with (a) $w = 0.01; 0.1; 1$ (b) $w = 1; 10; 100$; the bandwidth increases and the rise time decreases with w.

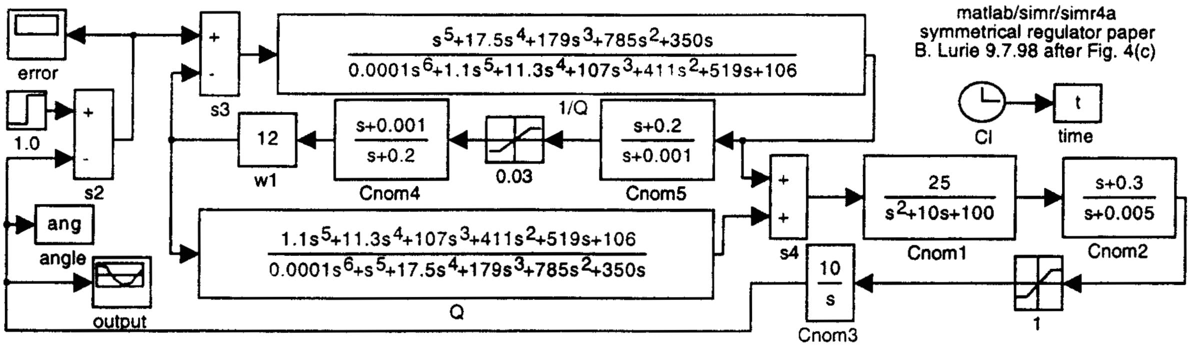


Fig. 12 SIMULINK model of proposed nonlinear controller

Thus, by varying only one real parameter in the compensator, the system loop shaping can be gradually and continuously changed from that optimal for acquisition to that optimal for tracking, and the intermediate responses seemingly suit the goal of fast and safe transition to tracking without loosing the acquisited target (signal). The time-profile of changing w can be optimized for specific properties of the signal to be tracked, or just safely chosen to be sufficiently shallow.

8. Nonlinear controller

As described, the adaptive one-parameter controller can be modified to work as single-nonlinear element nonlinear controller. With this Q and with the block w implemented as a link with saturation having 0.02 threshold, the regulator was implemented using block diagram shown in Fig. 4(c). The SIMULINK ® model of a system with this compensator and plant is shown in Fig. 12. The parameters of the links in the regulating element path have been found by trial-and-error procedure. A better design procedure for these links is yet to be developed.

The transient response to step input of different amplitudes is shown in Fig. 13. The working range of good performance is for the errors up to 1. It is seen that the acquisition transient response (response to large errors) is fast, much faster than the response to small error (the tracking response). It is also seen that exceeding this range does not lead to a catastrophic failura.

Conclusion

It was demonstrated that using nonlinear dynamic compensation based on Bode symmetrical regulator allows achieving simultaneously good tracking and good acquisition.

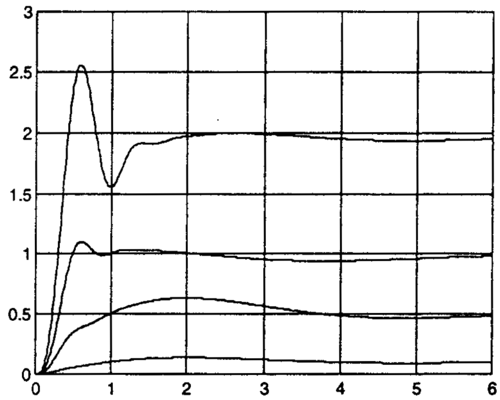


Fig. 13 Transient responses of the homing system with saturation link in the regulator, with step commands 0.1; 0.3; 1; 2.

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